MULTI-CRITERIA RANKING INDEX

Hong T. Tu [1], **Khu P. Nguyen** [2,*]

[1] HCMC University of Technology and Education
[2] HCM City University of Foreign Languages – Information Technology
hongtt@hcmute.edu.vn, khunp@huflit.edu.vn

ABSTRACT— Decision-making is sometimes complex and difficult especially when it involves ranking alternatives in the presence of multiple, usually conflicting, criteria. So far, TOPSIS, an acronym of Technique for Order Preference by Similarity to an Ideal Solution, has been used to select an alternative that should have the smallest distance from the benefit criteria and the furthest distance from the cost criteria. So far, this method is frequently used for making decisions in a variety of real-world situations. To take interest in the importance of alternatives in ranking, this paper proposes an improved technique to the original TOPSIS built on credit scores of alternatives. Experimental results of the paper show that the suggested technique is acceptable and appropriate for the requirements of the decision-making problem.

Keywords— Multi-criteria Decision making, TOPSIS, Ranking index, Complex network, Spearman rank-correlation.

I. INTRODUCTION

The goal of the decision-making problem is to identify a superior alternative among all viable alternatives. This problem is sometimes a complex and challenging task, especially when it involves finding the best alternative or ranking all alternatives in the presence of multiple, usually conflicting, criteria. To deal with these difficulties the decision-makers need to solve the MCDM (multi-criteria decision making) problem. There are various methods to solve for MCDM, one of them introduced by Hwang in [1] is known as a TOPSIS. The core concept of this technique is that the chosen alternative should have the smallest geometrical distance from the benefit criteria or the positive ideal solution (PIS) and the largest geometrical distance from the cost criteria the negative ideal solution (NIS) [2].

Since then, this technique has been widely applied in the past decades with satisfactory results in different fields of life such as energy [3], education [4], medicine [5], engineering and manufacturing systems [6], safety and environmental fields [7] chemical engineering [8], water resources studies [9], biology, diagnosis of disease [10,11,12], and so on.

To apply the original TOPSIS [1,2], it is required a specification of a set of alternatives $A = \{a_i \mid i=1, \ldots, m\}$, a set of criteria $C = \{c_j \mid j=1, \ldots, n\}$, let $X = \{x_{ij} \mid i=1, \ldots, m; j=1, \ldots, n\}$ denote the decision matrix where x_{ij} is the performance measure of a_i with respect to the criteria c_j, and a set of weights $W = \{w_j \mid j=1, \ldots, n\}$, $wj > 0$ and $\sum_{j=1\ldots n} w_j = 1$, where w_j denotes the weight of the criteria c_j.

The procedure of the original TOPSIS consists of the six steps: (i) Normalize the decision matrix X using Euclidean distance, (ii) Compute the weighted normalized decision matrix with W, (iii) Determine the set of benefit criteria or PIS and the set of cost criteria or NIS, (iv) Calculate the separation measures using the n-dimensional Euclidean distance of an alternative from the PIS and the NIS, (v) Calculate the relative closeness of an alternative to the PIS, named ranking index and (vi) Arrange the ranking indexes in a descending order to obtain the best alternative. The steps of the original TOPSIS seem reasonable. However, the relative importance of the distance from NIS does not consider, while the distances of an alternative from both the PIS and the NIS are major concerns in realistic decision-making. In addition, the original TOPSIS ignored attributes of alternatives. In this paper, such attributes are factors to describe operative credit scores which measure the importance of alternatives.

The remainder of the paper includes the following sections. Section II illustrates a brief literature review on the TOPSIS applications. Section III presents the ranking index and credit scores of alternatives. The next section illustrates some case studies of the problem. The paper ends with conclusions.

II. LITERATURE REVIEW

Firstly, the authors in [13] introduced methods of decision adding, namely TOPSIS and VIKOR (Vise Kriterijumska Optimizacija I Kompromisno Resenje, the Serbian name, means Multi-criteria Optimization and Compromise Solution). The VIKOR algorithm is based on ranking and selecting from a set of alternatives under conflicting criteria. By comparing the distances to the ideal alternative, this algorithm outputs the result of compromise ranking. The VIKOR with improvements is frequently used for ranking the online education

* Corresponding Author

programs in business administrations. The fact-finding results demonstrated that the VIKOR algorithm is remarkably successful to determine the best programs. In addition, the sensitivity analysis of the proposed method can use for monitoring the overall performance of the schools and to determine the strengths and weaknesses of the online business programs [14].

Company ranking is a complex process in which multiple financial ratios need to consider simultaneously. To make a company ranking under the real-time financial environment, the authors in [15] applied TOPSIS together with the method of experiment design. Case studies and results of the paper proved that the applicability, potentiality, simplicity, and flexibility of this method in making company ranking.

In [16], the authors used TOPSIS to evaluate the quality credit of the enterprises. Based on a set of data from eight air-conditioning enterprises, TOPSIS provides an effective method to determine which air-conditioning enterprises are high-performance. The analysis of realistic situations verified that the proposed quality credit indicator system is dependable and TOPSIS is suitable for quality credit evaluation.

To compare algorithms performance, the authors in [17] proposed a novel method based on TOPSIS to solve the problem of ranking and comparing the performance of algorithms. In this study, the alternatives consist of the algorithms and the criteria are the algorithm benchmarks. The simulation results in [17] showed the feasibility of this application to determine the ranking of algorithms.

In [18], the authors presented a method to rank the network node features based on TOPSIS. This study used centrality measures in a complex network together with solutions of the epidemic models [19] as weights to estimate the spreading ability of the top-ranked nodes. The experimental results on real networks indicated that their method attained a better performance to identify influential nodes.

In project management, the selection of the best alternative has attracted increasing attention due to the uncertain environment with vague variables. In [20], the authors introduced a method of measuring the similarity of vague sets and using TOPSIS to make decisions in project management. Computational examples of this method proved that vague TOPSIS is one of the powerful methods to solve this problem. In addition, surveys in [21] offered a general view on developments of fuzzy TOPSIS methods and explored the use of fuzzy models in decision-making with multi-criteria. Recently, Zulqarnain et al. [12] used the trapezoidal fuzzy numbers or typical characteristics of humans to identify diseases [11,12]. The authors also developed the graphical model of the TOPSIS method for the selection of a medical clinic for the diagnosis of disease [10]. Further description of the fuzzy TOPSIS can find in [22,23].

III. TOPSIS RANKING INDEX WITH SCORES

Based on achievements were made public through [24], the accordance between computations of information spreading densities with TOPSIS ranking among centrality measures of social network nodes has motivated in adding the credit scores of alternatives to the original TOPSIS.

Given a set of alternatives $A = \{a_i \mid i = 1, 2, ..., m\}$, a set of criteria $C = \{c_j \mid j = 1, 2, ..., n\}$, and $X = (x_{ij})$ be the $m\times n$ decision matrix, here $x_{ij} \geq 0$ is the performance factor of the alternative a_i with respect to the criteria c_j. Let $s_i > 0$ denotes the importance of each alternative, each s_i is the operative credit score of a_i, where $S = \{s_j \mid i = 1, 2, ..., m\}$ is the set of m credit scores. Our proposed TOPSIS, named scored TOPSIS, consists of the following steps:

i. Normalize the set of credit scores S for m credit scores of alternatives:

$$\sigma_i = s_i / \textstyle\sum_{k=1...m} s_k,\ i = 1, 2, ..., m. \tag{1}$$

ii. Normalize the decision matrix X with respect to all criteria:

$$y_{ij} = x_{ij} / \textstyle\sum_{i=1...m} x_{ij},\ i = 1, 2, ..., m;\ j = 1, 2, ..., n. \tag{2}$$

iii. Match the attribute to each credit score in S:

$$\varepsilon_j = \textstyle\sum_{k=1...m} (1/|y_{kj} - \sigma_k|),\ j = 1, 2, ..., n. \tag{3}$$

iv. Evaluate the weight of each criterion:

$$\omega_j = \varepsilon_j / \textstyle\sum_{k=1...m} \varepsilon_k,\ j = 1, 2, ..., n. \tag{4}$$

v. Set up the weighted decision matrix $Z = (z_{ij})$ using the weights ω_j and the normalized entries y_{ij}:

$$z_{ij} = \omega_j y_{ij} / (\textstyle\sum_{k=1...m} y_{kj}^2)^{1/2},\ i = 1, 2, ..., m;\ j = 1, 2, ..., n. \tag{5}$$

vi. Determine the ideal benefit solution or PIS and the ideal cost solution or NIS:

$$A^+ = \{a_1^+, ..., a_j^+, ..., a_n^+\},\ \text{where } a_j^+ = \max_{i=1...m} z_{ij} \text{ if } j\in J_b \text{ or } \min_{i=1...m} z_{ij} \text{ if } j\in J_c \tag{6}$$

$$A^{-} = \{a_1^{-}, ..., a_j^{-}, ..., a_n^{-}\}, \text{ where } a_j^{-} = \min_{i=1...m} z_{ij} \text{ if } j\in J_b \text{ or } \max_{i=1...m} z_{ij} \text{ if } j\in J_c \quad (7)$$

Where, J_b is the set of benefit criteria and J_c is the set of cost criteria.

vii. Calculate the distances of an alternative a_i from the benefit or cost criteria given respectively by

$$D_i^{+} = \{ \sum_{j=1...m} (a_j^{+} - z_{ij})^2 \}^{1/2}, i = 1, 2, \ldots, m, \quad (8)$$

$$D_i^{-} = \{ \sum_{j=1...m} (a_j^{-} - z_{ij})^2 \}^{1/2}, i = 1, 2, \ldots, m. \quad (9)$$

viii. Calculate the relative closeness of a_i to the benefit or cost solution given respectively by:

$$RC_i^{+} = D_i^{+} / (D_i^{+} + D_i^{-}) \quad (10)$$

$$RC_i^{-} = D_i^{-} / (D_i^{+} + D_i^{-}) \quad (11)$$

In which $0 \le RC_i^{-}, RC_i^{+} \le 1$, each of these indexes is the overall or composite performance score of alternative a_i. These indexes denote the order preferences according to the ideal solutions defined in (6) and (7) respectively. The alternatives with lower RC_i^{+} are supposed to be more important and should be a higher priority, while the alternatives with higher RC_i^{-} are less important and lower priority. Because of using (10) and (11) are equivalent, that why the original TOPSIS used only the index in (11) for ranking alternatives.

It is noticed that in the original TOPSIS the set of credit scores S is not involved, so the weight of each criterion must be initially given by $w_j > 0$, j=1, …, n, with $\sum_{j=1...n} w_j = 1$ as described in Section I instead of calculating all weights ω_j, j=1, …, n, in (4) with operative credit scores, and the performance distance in (2) is normalized by using Euclidean distances.

The ranking index of TOPSIS techniques seems reasonable. However, the relative importance of the distance in (10) or (11) to whether the benefit or cost solution has still not been considered and it is remained an open question, although ranking with one of these indexes is still correct.

To conduct an intensive analysis on whether the ranking index of the original TOPSIS is reasonable, by relying on the original TOPSIS, in [25] Ting Kuo considered realistic situations to answer this question and proved that the ranking index of the original TOPSIS is quite irrespective of these two indexes, so the ranking results would not differ as to if the decision-maker has no preference for these indexes.

This flaw will certainly limit the applicability of TOPSIS. To address this problem, Ting Kuo suggested that the ranking index of an alternative a_i need to unify both the distances D_i^{+} and D_i^{-} by using two weights w^{+} and w^{-} of the benefit criteria and the cost criteria to evaluate the relative importance of the distances, where $0 < w^{+}, w^{-} < 1$ and $w^{+} + w^{-} = 1$, as follows:

$$RC_i = w^{+} D_i^{-} / \sum_{k=1...m} D_k^{-} - w^{-} D_i^{+} / \sum_{k=1...m} D_k^{+} \quad (12)$$

Where, $-1 \le RC_i \le 1$, i=1, 2, …, m, and the values of RC_i are arranged in a descending order to obtain the best alternative. Although in [25] RC_i is calculated with D_k^{-} and D_k^{+} of the original TOPSIS, it can also apply with (8), (9) in the scored TOPSIS.

IV. EXPERIMENTS

A. SCORED TOPSIS IN SOCIAL NETWORKS

Centrality measures, like degree, betweenness, closeness, and eigenvalue centrality, are used to identify important roles of nodes in complex networks [26]. In this experiment, these measures notate respectively by Dc, Cc, Bc, and Ec. TOPSIS can use as a trade-off among the centrality measures to evaluate influential nodes in the face of information spreading in complex networks.

A network of nine nodes illustrated in [24] is used to demonstrate this case. Using formulations in [26] the centrality measures Dc, Cc, Bc, and Ec are calculated with each node in Figure 1 as given in Table 1. Let these measures be four criteria, the network nodes play the alternatives in Table 1. The rightmost column in the table lists the index ranking RC according to the original TOPSIS.

The information densities F_k, k=1, 2, 3, at network nodes computed through three case studies in [24], as shown in Table 2, are used to define three sets of scores in computations with the scored TOPSIS. The Spearman's test of rank correlations can be used to check the appropriate between TOPSIS ranking RC and the density ranking RC_{Fk} in network nodes.

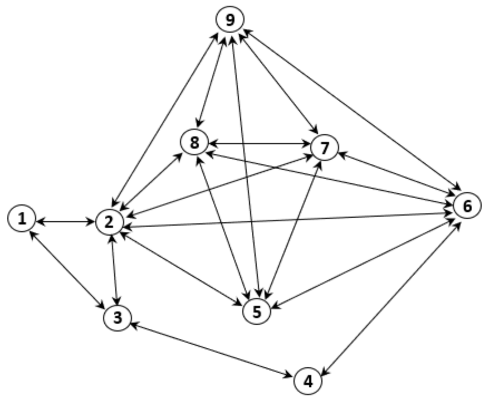

Fig. 1. *Network of nine nodes in [24]*

Table 1. *Centrality measures of the network in Fig. 1, where RC is the ranking of nodes by using TOPSIS*

i	Dc	Cc	Bc	Ec	RC
1.	2	0.571	0.000	0.108	9
2.	7	0.889	0.339	0.429	1
3.	3	0.615	0.054	0.125	7
4.	2	0.571	0.018	0.104	8
5.	5	0.727	0.000	0.391	4.5
6.	6	0.800	0.161	0.408	2
7.	5	0.727	0.000	0.391	4.5
8.	5	0.727	0.000	0.391	4.5
9.	5	0.727	0.000	0.391	4.5

Note: 4.5 is the middle order between the ranks 3 and 6

In Table 2, Spearman's test based on the statistic R_k [27] is defined as follows:

$$R_k = 1 - 6 \sum_{i=1\ldots n} d_{ki}^2/[n(n\text{-}1)] \tag{13}$$

Here, n is the number of network nodes, d_{ki} is the difference between the rank RC_{Fk} and the rank RC listed in Table 1 for each node i. The calculated values R_k show in the bottom row in Table 2.

Table 2. *The accordance between centralities and information diffusion spreading*

i.	RC	F1	RC_{F1}	F2	RC_{F2}	F3	RC_{F3}
1.	9	0.002	6	0.115	4	0.000	1
2.	1	0.334	9	0.001	1	0.196	6
3.	7	0.002	7	0.094	3	0.449	8
4.	8	0.000	1	0.137	5	0.001	2
5.	4.5	0.001	3	0.366	8.5	0.005	3
6.	2	0.223	8	0.005	2	0.670	9
7.	4.5	0.002	4.5	0.251	6.5	0.120	4.5
8.	4.5	0.000	2	0.366	8.5	0.240	7
9.	4.5	0.002	4.5	0.251	6.5	0.120	4.5
		R_1 = -0.388		R_2 = 0.250		R_3 = -0.529	

The Spearman's critical values at levels of significance 1%, 5%, 10%, are 0.834, 0.7, 0.6 [27], respectively. In Table 2, all the absolute values of R_k, k = 1, 2, 3, are not greater than these above critical values. Therefore, the original TOPSIS ranking RC of the nodes concerning their centrality measures is under each ranking F_k of information spreading density.

This result proved that each F_k can use as the set of scores of network nodes. Relying on these results, this paper proposed the scored TOPSIS.

B. SCORED TOPSIS IN SCHOOL SELECTION

In [28], a bunch of data from Summer Schools in Germany was collected in 2018. This data set consists of fifty-four universities with two non-beneficial criteria desired lower value are C_1: cost per duration, C_2: cost per credit; and three beneficial criteria desired higher values C_3: course curriculum, C_4: social and leisure, C_5: extra services valued from 1 to 10 corresponding to the lowest to highest benefit. A problem posed to the dataset: which one of these universities is the best Summer School based on specified criteria?

Due to the paper space, in this study, only the first ten universities in [28] are selected in a sample to demonstrate how to find the best Summer School based on the scored TOPSIS analysis. Table 3 illustrates this sample for the scored TOPSIS computation.

Table 3. *List of the first ten universities in [28]*

	University	C_1	C_2	C_3	C_4	C_5
1.	Bauhaus-Universität Weimar (a)	291.7	125	6	8	8
2.	Bauhaus-Universität Weimar (b)	291.7	166.7	6	8	8
3.	European University Viadrina Frankfurt (Oder) (a)	311.8	49	6	7	7

4.	European University Viadrina-Frankfurt (Oder) (b)	313.4	164.2	6	7	7
5.	Freie Universität Berlin	225	225	8	6	7
6.	Goethe-Universität Frankfurt (a)	560	333.3	7	8	8
7.	Goethe-Universität Frankfurt (b)	560	333.3	7	8	8
8.	Heidelberg University	781.7	167.5	7	6	7
9.	Heinrich-Heine-Universität Düsseldorf	140	125	6	6	6
10.	Humboldt-Universität zu Berlin (a)	238	121.4	6	7	7

The weighted decision matrix Z, the benefit solution A^+ and the cost solution A^- corresponding to the criteria are calculated by using (1)-(7) and illustrated in Figure 2. The relative closeness RC_i^+ or RC_i^- of each alternative a_i to the benefit solution or the cost solution computed with (10) (11) is illustrated behind the commas in the column RC^+ or RC^- of Table 4. The number before each comma is the rank of RC_i^+, RC_i^-. Hence, the alternative numbered 1 attains the first rank in the RC^+ column because this alternative is nearest to the benefit criteria and farthest to the cost criteria in the RC^- column, thus Bauhaus-Universität Weimar (a) University is the best Summer School to be selected.

While the eighth alternative or Humboldt-Universität zu Berlin (a) University ranked 10 in the RC^+ column is the worst selection because this alternative is farthest to the benefit criteria and nearest to the cost criteria in the RC^- column. In comparison with the data in the rows numbered 6 and 7 of Table 3 and the ones in Table 4, it is obvious that the rank of the 6th and 7th universities are the same.

Figure 2. *The weighted decision matrix with the benefit and cost solutions*

$$
Z = \begin{bmatrix}
0.0117 & 0.0166 & 0.0549 & 0.1455 & 0.0909 \\
0.0117 & 0.0221 & 0.0549 & 0.1455 & 0.0909 \\
0.0125 & 0.0065 & 0.0549 & 0.1273 & 0.0795 \\
0.0125 & 0.0218 & 0.0549 & 0.1273 & 0.0795 \\
0.0090 & 0.0298 & 0.0732 & 0.1091 & 0.0795 \\
0.0224 & 0.0442 & 0.0640 & 0.1455 & 0.0909 \\
0.0224 & 0.0442 & 0.0640 & 0.1455 & 0.0909 \\
0.0312 & 0.0222 & 0.0640 & 0.1091 & 0.0795 \\
0.0056 & 0.0166 & 0.0549 & 0.1091 & 0.0681 \\
0.0095 & 0.0161 & 0.0549 & 0.1273 & 0.0795
\end{bmatrix}
$$

$$
\begin{array}{llllll}
A^+: & 0.0056 & 0.0065 & 0.0732 & 0.1455 & 0.0909 \\
A^-: & 0.0312 & 0.0442 & 0.0549 & 0.1091 & 0.0681
\end{array}
$$

Table 4. *Scored TOPSIS ranking indexes*

i.	RC^+	RC^-	RC_a	RC_b
1.	1, 0.2847	10, 0.7153	1, 0.0210	1, -0.0083
2.	2, 0.3226	9, 0.6774	2, 0.0174	2, -0.0120
3.	3, 0.3803	8, 0.6197	3, 0.0116	3, -0.0175
4.	5, 0.4751	6, 0.5249	5, 0.0022	5, -0.0242
5.	9, 0.5677	2, 0.4323	9, -0.0068	8, -0.0370
6.	6, 0.4858	4, 0.5142	6, 0.0016	6, -0.0317
7.	6, 0.4858	4, 0.5142	6, 0.0016	6, -0.0317
8.	10, 0.6518	1, 0.3482	10, -0.0147	10, -0.0437
9.	8, 0.5586	3, 0.4414	8, -0.0064	9, -0.0391
10.	4, 0.4199	7, 0.5801	4, 0.0073	4, -0.0201

Similarly, the column RC_a in Table 4 describes the relative closeness RC_i of the alternative a_i calculated by (12) with $w^+ = w^- = 0.5$. Although the values of index ranking in the column RC^+ differ from the RC_a one the ranks in these columns are the same, this result is appropriate to the remark of Ting Kuo in [25]. However, when a decision-maker decreases w^+ to 0.2 and increases w^- to 0.8 the closeness ranking in the RC_b column is the same as RC_a except that the ranks of the alternatives numbered 5 and 9 in the columns RC_a and RC_b are exchanged with each other.

V. CONCLUSION

This study proposes the scored TOPSIS as an attempt to develop the ranking index of the original TOPSIS by introducing the credit scores associated with the alternatives to weight the criteria. The sets of benefit criteria and cost criteria are also weighted by non-negative convex combinations chosen by decision-makers. Experimental results showed that this improvement is reasonable and appropriate with realistic requirements. The proposed analysis approach can be used for conducting a comprehensive analysis of multi-criteria decision-making methods to find a compromised solution. The proposed ranking index applies not only to problems that can be solved by the original TOPSIS but also to other modified versions of TOPSIS.

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CHỈ SỐ XẾP HẠNG ĐA TIÊU CHÍ

Từ Tuyết Hồng, Nguyễn Phi Khứ

TÓM TẮT— Việc ra quyết định đôi khi gặp khó khăn, phức tạp do phải xếp hạng những chọn lựa theo nhiều tiên chí có thể xung khắc nhau. TOPSIS được dùng để đưa ra một chọn lựa gần với các tiêu chí có lợi và tránh xa các tiêu chí bất lợi. Phương pháp này thường dùng trong việc ra quyết định trong nhiều tình huống thực tế. Bài báo này đề xuất phương pháp xếp hạng dựa trên TOPSIS kèm theo các trọng số tin tưởng đối với có các chọn lựa. Kết quả thực nghiệm từ bài báo chứng tỏ phương pháp đề xuất là chấp nhận được và phù hợp với những yêu cầu của bài toán ra quyết định.

Hong T. Tu received a M.Sc. degree in Computer Science from the Ho Chi Minh City University of Technology, Vietnam National University, HCMC. She has published more than 15 papers in refereed conferences and international venues. Her research interests are social network analysis, latent semantic analysis, paralleled and distributed computing. ID http://orcid.org/0000-0002-8635-4875

Khu P. Nguyen received his Ph.D. in Computational Fluid Mechanics from Vietnam Ministry of Education and Training, attained associate professor in the Mechanics from the State Council for Professorship in 2010. He has published about 65 papers in refereed journals and other international venues, done research successfully in more than 20 R&D projects. His research interests are applied mathematics, statistics and stochastic processes, fluid mechanic models in water environment, design and analysis algorithms, high performance computing, mining information system, complex network analysis. ID http://orcid.org/0000-0001-7509-3537